## Complex Projectile Motion

Simple Examples (vertical):
a) How long does it take for a baseball thrown at an initial velocity of $4 \mathrm{~m} / \mathrm{s}$ [up] to reach its max height?
b) How long is the flight of the ball if it hits the ground where it started? (same vertical level)
c) How long is the flight of the ball if it hits the ground 10 m below where it started?


Recall: When doing a projectile motion question, you must separate the vertical and horizontal components.

Ex: Calculate the initial vertical velocity of a projectile launched at $20 \mathrm{~m} / \mathrm{s}$ [40 above the horizontal]

Ex: A soccer ball is kicked at $10 \mathrm{~m} / \mathrm{s}$ [ $15^{\circ}$ above the horizontal] and lands on the same vertical level down field.
a) What is the max height of the ball?
b) How long is the ball in the air?
c) How far down field will the ball travel?


Ex: A projectile is launched at $20 \mathrm{~m} / \mathrm{s}$ [ $40^{\circ}$ above the horizontal]. The object lands on a 5 m tall house. How far away is the house?

## PHYSICS

## Complex Projectile Motion

## Practice Question

You are on the top of a 25 m tall building and are looking to throw a football to your friend in the next building. Your friend's building is 35 m tall. If you launch the ball with a velocity of $40 \mathrm{~m} / \mathrm{s}$ [ $30^{\circ}$ above the horizontal]:
a) How long does it take for the ball to reach your friend?
b) How far apart are the buildings?


Ex: In order to sink a free-throw in basketball you have to throw the basket ball so that it travels a horizontal distance of 4.5 m while landing in the hoop 3m above the ground. With what velocity must you throw the ball if you toss it at an angle of $60^{\circ}$ to the horizontal?

PHYSICS

## Complex Projectile Motion

## PROJECTILE MOTION REVIEW

3. A marble rolls off a table with a velocity of $1.93 \mathrm{~m} / \mathrm{s}$ [horizontally]. The tabletop is 76.5 cm above the floor. If air resistance is negligible, determine
(a) how long the marble is airborne
(b) the horizontal range
(c) the velocity at impact
4. A stone is thrown horizontally with an initial speed of $8.0 \mathrm{~m} / \mathrm{s}$ from a cliff. Air resistance is negligible.
(a) Determine the horizontal and vertical components of displacement and instantaneous velocity at $t=0.0 \mathrm{~s}, 1.0 \mathrm{~s}, 2.0 \mathrm{~s}$, and 3.0 s .
(b) Draw a scale diagram showing the path of the stone.
(c) Draw the instantaneous velocity vector at each point on your diagram.
(d) Determine the average acceleration between 1.0 s and 2.0 s , and between 2.0 s and 3.0 s . What do you conclude?
5. A baseball pitcher throws a ball horizontally under negligible air resistance.

The ball falls 83 cm in travelling 18.4 m to the home plate. Determine the ball's initial horizontal speed.
8. A field hockey ball is struck and undergoes projectile motion. Air resistance is negligible.
(a) What is the vertical component of velocity at the top of the flight?
(b) What is the acceleration at the top of the flight?
(c) How does the rise time compare to the fall time if the ball lands at the same level from which it was struck?
9. A cannon is set at an angle of $45^{\circ}$ above the horizontal. A cannonball leaves the muzzle with a speed of $2.2 \times 10^{2} \mathrm{~m} / \mathrm{s}$. Air resistance is negligible. Determine the cannonball's
(a) maximum height
(b) time of flight
(c) horizontal range (to the same vertical level)
(d) velocity at impact
10. A medieval prince trapped in a castle wraps a message around a rock and throws it from the top of the castle wall with an initial velocity of $12 \mathrm{~m} / \mathrm{s}$ [ $42^{\circ}$ above the horizontal]. The rock lands just on the far side of the castle's moat, at a level 9.5 m below the initial level (Figure 14). Determine the rock's
(a) time of flight
(b) width of the moat
(c) velocity at impact


Figure 14
The situation for question 10

## Answers

3. (a) 0.395 s
(b) 76.3 cm
(c) $4.33 \mathrm{~m} / \mathrm{s}\left[63.5^{\circ}\right.$ below the horizontal]
4. (a) At $3.0 \mathrm{~s}, \Delta x=24 \mathrm{~m}$, $\Delta y=44 \mathrm{~m}$, and $\vec{v}=3.0 \times 10^{1} \mathrm{~m} / \mathrm{s}$ [ $75^{\circ}$ below the horizontal].
(d) $9.8 \mathrm{~m} / \mathrm{s}^{2}$ [down]
5. $45 \mathrm{~m} / \mathrm{s}$

## Answers

9. (a) $1.2 \times 10^{3} \mathrm{~m}$
(b) 32 s
(c) $4.9 \times 10^{3} \mathrm{~m}$
(d) $2.2 \times 10^{2} \mathrm{~m} / \mathrm{s}\left[45^{\circ}\right.$ belo the horizontal]
10. (a) 2.4 s
(b) 22 m
(c) $18 \mathrm{~m} / \mathrm{s}\left[60^{\circ}\right.$ below the horizontal]

## PHYSICS

## PROJECTILE MOTION REVIEW

3. A projectile launched horizontally moves 16 m in the horizontal plane while falling 1.5 m in the vertical plane. Determine the projectile's initial velocity.
4. A tennis player serves a ball horizontally, giving it a speed of $24 \mathrm{~m} / \mathrm{s}$ from a height of 2.5 m . The player is 12 m from the net. The top of the net is 0.90 m above the court surface. The ball clears the net and lands on the other side.
Air resistance is negligible.
(a) For how long is the ball airborne?
(b) What is the horizontal displacement?
(c) What is the velocity at impact?
(d) By what distance does the ball clear the net?
5. A child throws a ball onto the roof of a house, then catches it with a baseball glove 1.0 m above the ground, as in
Figure 15. The ball leaves the roof with a speed of $3.2 \mathrm{~m} / \mathrm{s}$.
(a) For how long is the ball airborne after leaving the roof?
(b) What is the horizontal distance from the glove to the edge of the roof?
(c) What is the velocity of the ball just before it lands in the glove?

6. During World War I, the German army bombarded Paris with a huge gun referred to, by the Allied Forces, as "Big Bertha." Assume that Big Bertha fired shells with an initia velocity of $1.1 \times 10^{3} \mathrm{~m} / \mathrm{s}$ [ $45^{\circ}$ above the horizontal].
(a) How long was each shell airborne, if the launch point was at the same level as the landing point?
(b) Determine the maximum horizontal range of each she
(c) Determine the maximum height of each shell.
7. An astronaut on the Moon, where $|\vec{g}|=1.6 \mathrm{~m} / \mathrm{s}^{2}$, strikes golf ball giving the ball a velocity of $32 \mathrm{~m} / \mathrm{s}$ [ $35^{\circ}$ above the Moon's horizontal]. The ball lands in a crater floor that is 15 m below the level where it was struck. Determine
(a) the maximum height of the ball
(b) the time of flight of the ball
(c) the horizontal range of the ball
8. $29 \mathrm{~m} / \mathrm{s}$ [horizontally]
9. (a) 0.71 s
(b) 17 m
(c) $25 \mathrm{~m} / \mathrm{s}\left[16^{\circ}\right.$ below the horizontal]
(d) 0.38 m
10. (a) 0.87 s
(b) 2.3 m
(c) $11 \mathrm{~m} / \mathrm{s}\left[75^{\circ}\right.$ below the horizontal]
11. (a) $1.6 \times 10^{2} \mathrm{~s}$
(b) $1.2 \times 10^{5} \mathrm{~km}$
(c) 31 km
12. (a) $1.1 \times 10^{2} \mathrm{~m}$
(b) 24 s
(c) $6.2 \times 10^{2} \mathrm{~m}$

Figure 15

## PHYSICS

## Complex Projectile Motion

## Projectile Motion Worksheet

1) A ball rolls with a speed of $2.0 \mathrm{~m} / \mathrm{s}$ across a level table that is 1.0 m above the floor. Upon reaching the edge of the table, it follows a parabolic path to the floor. How far along the floor is the landing spot from the table? [ 0.90 m ]
2) A rescue pilot drops a survival kit while her plane is flying at an altitude of 2000.0 m with a forward velocity of $100.0 \mathrm{~m} / \mathrm{s}$. If air friction is disregarded, how far in advance of the starving explorer's drop zone should she release the package? [ 2020 m ]
3) A rifle is fired horizontally and travels $200.0 \mathrm{~m}[\mathrm{E}]$. The rifle barrel is 1.90 m from the ground. What speed must the bullet have been travelling at? Ignore friction. [ $321 \mathrm{~m} / \mathrm{s}$ ]
4) A skier leaves the horizontal end of a ramp with a velocity of $25.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$ and lands 70.0 m from the base of the ramp. How high is the end of the ramp from the ground? [ 38.5 m ]
5) An astronaut stands on the edge of a lunar crater and throws a half-eaten Twinkie ${ }^{\mathrm{TM}}$ horizontally with a velocity of $5.00 \mathrm{~m} / \mathrm{s}$. The floor of the crater is 100.0 m below the astronaut. What horizontal distance will the Twinkie ${ }^{\mathrm{TM}}$ travel before hitting the floor of the crater? (The acceleration of gravity on the moon is $1 / 6^{\text {hi }}$ that of the Earth). [ 55.3 m ]
6) A baseball player leads off the game and hits a long home run. The ball leaves the bat at an angle of $30.0^{\circ}$ from the horizontal with a velocity of $40.0 \mathrm{~m} / \mathrm{s}$. How far will it travel in the air? [ 141 m ]
7) Agolfer is teeing off on a 170.0 m long par 3 hole. The ball leaves with a velocity of $40.0 \mathrm{~m} / \mathrm{s}$ at $50.0^{\circ}$ to the horizontal. Assuming that she hits the ball on a direct path to the hole, how far from the hole will the ball land (no bounces or rolls)? $[9.38 \mathrm{~m}]$
8) A punter in a football game kicks a ball from the goal line at $60.0^{\circ}$ from the horizontal at $25.0 \mathrm{~m} / \mathrm{s}$.
a) What is the hang time of the punt? [4.41 s]
b) How far down fiel d does the ball land? [ 55.2 m ]
9) A cannon fires a cannonball 500.0 m downrange when set at a $45.0^{\circ}$ angle. At what velocity does the cannonball leave the cannon? [ $70.0 \mathrm{~m} / \mathrm{s}$ at $45.0^{\circ}$ ]

## PHYSICS

10) A lovesick lad wants to throw a bag of candy and love notes into the open window of his girlfriend's bedroom 10.0 m above. Assuming it just reaches the window, he throws the love gifts at $60.0^{\circ}$ to the ground:
a) At what velocity should she throw the bag? $\left[16.2 \mathrm{~m} / \mathrm{s}\right.$ at $60.0^{\circ}$ to the ground $]$
b) How far from the house is he standing when he throws the bag? [ 11.5 m ]
11) You are piloting a helicopter which is rising vertically at a uniform velocity of 14.70 $\mathrm{m} / \mathrm{s}$. When you reach 196.00 m , you see Barney (Uh-oh). Alarge object is projected with a horizontal velocity of $8.50 \mathrm{~m} / \mathrm{s}$ from the rising helicopter.
a) When does the ball reach Barney's head if he is standing in a hole with his head at ground level? [7.99 s]
b) Where does Barney have to be horizontally relative to the helicopter's position? [ 68.0 m ]
c) What is the vertical velocity when it hits the ground [ $[63.7 \mathrm{~m} / \mathrm{s}$ ]
12) An object is punted at $25.0 \mathrm{~m} / \mathrm{s}\left[40.0^{\circ} \mathrm{N}\right.$ of E$]$ on G 's home planet. What is the range of the object on level ground? (Useg $=18.0 \mathrm{~m} / \mathrm{s}^{2}$ ) $[34.2 \mathrm{~m}]$
13) An elastic loaded balloon launcher fires balloons at an angle of $\left[38.0^{\circ} \mathrm{N}\right.$ of E$]$ from the surface of the ground. If the initial velocity is $25.0 \mathrm{~m} / \mathrm{s}$, find how far away the balloons are from the launcher when they hit the level ground again. [ 61.8 m ]
14) A movie stunt driver on a motor cycle speeds horizontally off a 50.0 m high cliff. How fast (in km/h) must the motor cycle leave the cliff-top if it's to land on the level ground below at a distance of 90.0 m from the base of the cliff? [ $101 \mathrm{~km} / \mathrm{h}$ ]
15) A football is kicked at $37.0^{\circ}$ to the horizontal at $20.0 \mathrm{~m} / \mathrm{s}$ from the player's hand at 1.00 m from the ground. How far did the football travel before hitting the ground? [ 40.5 m ]
16) The same football in \#15 is kicked from the ground instead.
a) Find the maximum height. [ 7.38 m ]
b) Find the time of travel. [ 2.45 s ]
c) How far away does it hit the ground? [ 39.2 m ]
d) Find the velocity vector at maximum height. [ $16.0 \mathrm{~m} / \mathrm{s}$ which is horizontal]
e) Find the acceleration vector at maximum height. [ $9.81 \mathrm{~m} / \mathrm{s}^{2}$ down]
