

Equations of Motion

1.2

You are driving a car along a road at a constant speed. Suddenly, you see a deer crossing the road ahead of you (**Figure 1**). You apply your brakes, slowing the car down. If you slow down at a constant rate, how far will you travel before you stop? Will you hit the animal?

To solve this kind of problem, you can use motion graphs. Using these graphs, you can calculate the distance you would travel while applying the brakes, which is the braking distance. You can also determine your braking time. The braking time is how long it takes to stop while applying the brakes.

There is another way to solve the problem, though: using motion equations. Motion equations relate different variables such as velocity, acceleration, and displacement. In most cases, it is easier and faster to solve problems using motion equations than by drawing motion graphs.

For motion with constant acceleration, motion equations relate five variables: the object's initial velocity, its final velocity, its acceleration, its displacement, and the time interval.



Figure 1 Understanding the physics of motion can help you prevent accidents.

One-Dimensional Motion with Constant Acceleration

The defining equation for average acceleration is

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

In this section and for all our equations of motion, the acceleration is constant, so we will use the symbol \vec{a} for \vec{a}_{av} .

Figure 2 shows a velocity–time graph for an object with constant acceleration. The object starts with initial velocity \vec{v}_i , accelerates for a time Δt with acceleration \vec{a} , and ends with final velocity \vec{v}_f . The displacement is equal to the area under the line. The shape under the line is a trapezoid, and its area is

$$\Delta \vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$$

Notice that the above equation does not use the variable \vec{a} . This means we can combine it with the defining equation for average acceleration to derive other useful equations. There are five variables (Δt , $\Delta \vec{d}$, \vec{v}_i , \vec{v}_f , and \vec{a}), and we can derive equations that link any four of them.

For example, to eliminate \vec{v}_f , we can combine our equations. We can rearrange the defining equation for average acceleration to get \vec{v}_f :

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

Substituting the above equation for \vec{v}_f into the equation for displacement gives

$$\begin{aligned}\Delta \vec{d} &= \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t \\ &= \frac{1}{2}((\vec{v}_i + \vec{a}\Delta t) + \vec{v}_i)\Delta t\end{aligned}$$

$$\Delta \vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

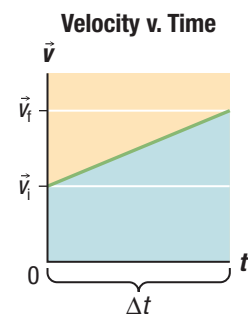


Figure 2 The displacement of an object is equal to the area under the line on a velocity–time graph. For constant acceleration, this shape is a trapezoid.

We can use similar substitutions to derive two more equations that eliminate \vec{v}_i and Δt . The equations of motion for constant acceleration are shown in **Table 1**. In the following Tutorial, you will solve motion problems using these kinematics equations.

UNIT TASK BOOKMARK

You can use some of the equations in Table 1 when you complete the Unit Task on page 146.

Table 1 The Five Key Equations for Uniformly Accelerated Motion

	Equation	Variables found in equation	Variable not in equation
Equation 1	$\Delta \vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2} \right) \Delta t$	$\Delta \vec{d}, \Delta t, \vec{v}_i, \vec{v}_f$	\vec{a}
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$	$\vec{v}_f, \vec{v}_i, \vec{a}, \Delta t$	$\Delta \vec{d}$
Equation 3	$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$	$\Delta \vec{d}, \vec{v}_i, \Delta t, \vec{a}$	\vec{v}_f
Equation 4	$v_f^2 = v_i^2 + 2a\Delta d$	$v_f, v_i, a, \Delta d$	Δt
Equation 5	$\Delta \vec{d} = \vec{v}_i \Delta t - \frac{1}{2} \vec{a} \Delta t^2$	$\Delta \vec{d}, \vec{v}_i, \Delta t, \vec{a}$	\vec{v}_f

Tutorial 1 Using the Equations of Motion to Solve for Accelerated Motion

In the following Sample Problems, we will use the kinematics equations to analyze motion.

Sample Problem 1: Solving for Time and Final Velocity

Two cars are at rest on a straight road. Car A starts 120 m ahead of car B, and both begin moving in the same direction at the same time. Car A moves at a constant velocity of 7.0 m/s [forward]. Car B moves at a constant acceleration of 2.0 m/s² [forward]. Calculate how long it will take for car B to catch up with car A, and calculate the velocities of the two cars when they meet.

Given: We have two sets of variables, one for each car.

We will label them with subscripts A and B: \vec{v}_{Ai} ; \vec{v}_{Bi} ; \vec{a}_A ; \vec{a}_B ;

$\vec{v}_{Ai} = 7.0$ m/s [forward]; $\vec{a}_A = 0$ m/s²; $\vec{v}_{Bi} = 0$ m/s;

$\vec{a}_B = 2.0$ m/s² [forward]

Required: Δt ; \vec{v}_{Af} ; \vec{v}_{Bf}

Analysis: Using $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$, express the displacement of each car using forward as positive, and then relate the two displacements.

Solution: The displacement of car A from its starting position is

$$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\Delta \vec{d}_A = (7.0 \text{ m/s [forward]}) \Delta t + \left(\frac{1}{2} (0) \right) \Delta t^2$$

$$\Delta \vec{d}_A = (7.0 \text{ m/s [forward]}) \Delta t$$

The displacement of car B from its starting position is

$$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\Delta \vec{d}_B = (0) \Delta t + \frac{1}{2} (2.0 \text{ m/s}^2 \text{ [forward]}) \Delta t^2$$

$$\Delta \vec{d}_B = (1.0 \text{ m/s}^2 \text{ [forward]}) \Delta t^2$$

We want to solve for the time at which car B has caught up with car A. That is when $\Delta \vec{d}_B = \Delta \vec{d}_A + 120$ m [forward] because, for car B to catch up, it has to cover the initial 120 m separation as well as the displacement of car A.

Using our expressions for $\Delta \vec{d}_A$ and $\Delta \vec{d}_B$, we get the following quadratic equation:

$$\Delta \vec{d}_B = \Delta \vec{d}_A + 120 \text{ m}$$

$$\Delta t^2 = 7.0 \Delta t + 120$$

Note that we are omitting the units here to keep the equation uncluttered. Rearranging gives the equation below.

$$\Delta t^2 - 7.0 \Delta t - 120 = 0$$

$$(\Delta t - 15)(\Delta t + 8.0) = 0$$

There are two solutions: $\Delta t = 15$ and $\Delta t = -8.0$. We want the positive time value, which is $\Delta t = 15$ s.

Since car A is not accelerating, the velocity of car A is unchanged: $\vec{v}_{Af} = 7.0$ m/s [forward].

We can determine the final velocity of car B:

$$\vec{v}_{Bf} = \vec{v}_{Bi} + \vec{a}_B \Delta t$$

$$= 0 + (2.0 \text{ m/s}^2 \text{ [forward]}) (15 \text{ s})$$

$$\vec{v}_{Bf} = 3.0 \times 10^1 \text{ m/s [forward]}$$

Statement: Car B catches up with car A after 15 s. When they meet, the velocity of car A is 7.0 m/s [forward], and the velocity of car B is 3.0×10^1 m/s [forward].

Sample Problem 2: Solving for Time and Displacement

A motorcyclist drives along a straight road with a velocity of 30.0 m/s [forward]. The driver applies the brakes and slows down at 5.0 m/s² [backward].

- (a) Calculate the braking time.
(b) Determine the braking distance (displacement).

Solution

- (a) **Given:** $\vec{v}_i = 30.0 \text{ m/s}$ [forward]; $\vec{a} = 5.0 \text{ m/s}^2$ [backward];
 $\vec{v}_f = 0$

Required: Δt

Analysis: $a = \frac{v_f - v_i}{\Delta t}$; the motorcycle is slowing down, so make the acceleration negative and the initial velocity positive.

Solution: $a = \frac{v_f - v_i}{\Delta t}$

$$\begin{aligned}\Delta t &= \frac{v_f - v_i}{a} \\ &= \frac{0 \text{ m/s} - 30.0 \text{ m/s}}{-5.0 \text{ m/s}^2} \\ \Delta t &= 6.0 \text{ s}\end{aligned}$$

Statement: The braking time is 6.0 s.

- (b) **Given:** $\vec{v}_i = 30.0 \text{ m/s}$ [forward]; $\vec{a} = -5.0 \text{ m/s}^2$ [forward];
 $\Delta t = 6.0 \text{ s}$

Required: $\Delta \vec{d}$

Analysis: $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$

Solution:

$$\begin{aligned}\Delta \vec{d} &= \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2 \\ &= (30.0 \text{ m/s} [\text{forward}])(6.0 \text{ s}) + \\ &\quad \frac{1}{2} (-5.0 \text{ m/s}^2 [\text{forward}])(6.0 \text{ s})^2 \\ &= 180.0 \text{ m} [\text{forward}] - 90.0 \text{ m} [\text{forward}] \\ \Delta \vec{d} &= 90.0 \text{ m} [\text{forward}]\end{aligned}$$

Statement: The braking distance is 90.0 m [forward].

Practice

- A motorcyclist is travelling at 15.0 m/s [forward] and applies the brakes. The motorcycle slows down at 5.0 m/s² [backward]. **T/I A**
 - Determine the motorcycle's braking distance. [ans: 22 m [forward]]
 - Compare your answer to (a) to the answer for Sample Problem 2. What do these two problems indicate about speeding and traffic safety?
- A man starts at rest and then runs north with a constant acceleration. He travels 120 m in 15 s. Calculate his acceleration. **T/A** [ans: 1.1 m/s² [N]]
- A bus is moving at 22 m/s [E] for 12 s. Then the bus driver slows down at 1.2 m/s² [W] until the bus stops. Determine the total displacement of the bus. **T/A** [ans: 4.7 × 10² m]
- In a 100.0 m sprint, a runner starts from rest and accelerates to 9.6 m/s [W] in 4.2 s. **T/I**
 - Calculate the acceleration of the runner. [ans: 2.3 m/s² [W]]
 - Calculate the displacement of the runner. [ans: 2.0 × 10¹ m [W]]
 - The runner runs at a constant velocity for the rest of the race. What is the total time? [ans: 13 s]
- Two football players separated by 42 m run directly toward each other. Football player 1 starts from rest and accelerates at 2.4 m/s² [right], and football player 2 moves uniformly at 5.4 m/s [left]. **K/U T/I A**
 - How long does it take for the players to collide? [ans: 4.1 s]
 - How far does each player move? [ans: player 1: 20 m; player 2: 22 m]
 - How fast is football player 1 moving when the players collide? [ans: 9.8 m/s]
- A jet lands on a runway at 110 m/s [forward]. When stopping, the jet can accelerate at 6.2 m/s² [backward]. **K/U T/I A**
 - Calculate the minimum time for the jet to stop. [ans: 18 s]
 - What is the minimum safe length for this runway? [ans: 9.8 × 10² m]
 - Explain why the runway should be much longer than the minimum safe length.

Freely Falling Objects

free fall the motion of a falling object where the only force acting on the object is gravity

When you let go of a ball, it will fall down because Earth's gravity pulls it down. When you throw a ball upward, it will move upward for a time, stop, change direction, and then move downward. In both cases, the ball is moving under the influence of gravity. An object that is only moving under the influence of gravity is said to be in **free fall**. The ball is in free fall when you drop it, and the ball is in free fall when you throw it upward. Gravity moves objects downward with a constant acceleration of 9.8 m/s^2 . This important value is called the acceleration due to gravity, g .

On Earth, other factors affect moving objects, such as air resistance. You will read more about air resistance in Chapter 2.

Tutorial 2 Using the Equations of Motion for Accelerated Motion Due to Gravity

Sample Problem 1: Calculating Time and Velocity

A ball is thrown from a height of 52 m from the top of a building with a velocity of 24 m/s straight up.

- Determine the velocity of the ball at ground level.
- How long does it take for the ball to reach the ground?

Solution

- (a) **Given:** $\vec{v}_i = 24 \text{ m/s}$ [up]; $\vec{g} = 9.8 \text{ m/s}^2$ [down];

$$\Delta \vec{d} = 52 \text{ m} \text{ [down]}$$

Required: \vec{v}_f

Analysis: Use $v_f^2 = v_i^2 + 2a\Delta d$ to calculate the final velocity. Use up as positive and down as negative.

Solution: $v_f^2 = v_i^2 + 2a\Delta d$

$$v_f^2 = (+24 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(-52 \text{ m})$$

$$v_f = \pm 39.9 \text{ m/s} \text{ (one extra digit carried)}$$

Since the ball is moving down, use the negative value.

Statement: The velocity of the ball at ground level is $-4.0 \times 10^1 \text{ m/s}$, or $4.0 \times 10^1 \text{ m/s}$ [down].

- (b) **Given:** $\vec{v}_i = +24 \text{ m/s}$; $\vec{v}_f = -39.9 \text{ m/s}$; $\vec{g} = -9.8 \text{ m/s}^2$

Required: Δt

$$\textbf{Analysis: } \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

$$\textbf{Solution: } \Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

$$= \frac{(-39.9 \text{ m/s}) - (+24 \text{ m/s})}{-9.8 \text{ m/s}^2}$$

$$\Delta t = 6.5 \text{ s}$$

Statement: It takes 6.5 s for the ball to reach the ground.

Practice

- An apple is thrown upward from ground level at 22 m/s. Determine the maximum height of the apple. **T/I** [ans: 25 m]
- Calculate the length of time it would take an object to fall 10.0 m if g were one-sixth the value of Earth's g (the acceleration due to gravity on the Moon). **T/I** [ans: 3.5 s]
- A ball is thrown straight down at 12 m/s toward the ground from a height of 45 m. **T/I**
 - How long does it take to land? [ans: 2.0 s]
 - How fast is the ball moving when it lands? [ans: 32 m/s]
 - How does your answer to (b) change if the ball is initially thrown at 12 m/s [upward]?
- In a physics experiment, a ball is released from rest, and it falls toward the ground. The timer was not paying attention but estimates that it took 1.5 s for the ball to fall the last 32 m. **T/I**
 - Calculate the velocity of the ball when it is 32 m above the ground. [ans: 14 m/s [downward]]
 - Calculate the total displacement of the ball. [ans: 42 m [downward]]
- Matt tosses a set of keys straight up to Sanjit, who catches them from a window 1.1 s later at a height of 14 m above the point of release. **T/I**
 - Determine the initial velocity of the keys. [ans: 18 m/s [up]]
 - Determine the velocity of the keys when they were caught. [ans: 7.3 m/s [up]]

1.2 Review

Summary

- The five key equations for uniformly accelerated motion, listed in Table 1 on page 18, use the following variables: displacement, initial velocity, final velocity, acceleration, and time interval.
- Free fall is the motion of an object when it is moving only under the influence of gravity.
- The acceleration due to gravity on Earth is 9.8 m/s^2 [down].

Questions

- Upon leaving the starting gate, a racehorse accelerates at a constant 4.1 m/s^2 [forward] for 5.2 s. Determine the horse's
 - displacement
 - final velocity **T/I A**
- An electron travelling at $7.72 \times 10^6 \text{ m/s}$ [E] enters a force field that reduces its velocity to $2.46 \times 10^6 \text{ m/s}$ [E]. The acceleration is constant. The displacement during the acceleration is 0.478 m [E]. **T/I A**
 - Determine the electron's acceleration.
 - Determine the time interval over which the acceleration occurs.
- A police officer at rest at the side of the highway notices a speeder moving at 62 km/h along a straight level road near an elementary school. When the speeder passes, the officer accelerates at 3.0 m/s^2 in pursuit. The speeder does not notice until the police officer catches up. **T/I A**
 - How long will it take for the officer to catch the speeder?
 - How far will they move from the position where the officer was at rest?
 - Calculate the speed of the police car when the officer catches the speeder. Is this reasonable?
 - Now assume that the police officer accelerates until the police car is moving 10.0 km/h faster than the speeder and then moves at a constant velocity until the police officer catches up. How long will it take to catch the speeder? Is this scenario more reasonable than the scenario in (c)? Explain your answer.
- A cliff rises straight up from the Mediterranean Sea. Explain how you could calculate the height of the cliff above the sea using a stopwatch and a small stone. What assumption must you make? **K/U T/I C A**
- A baseball player throws a ball straight up and then catches it 2.4 s later at the same height from which he threw it. Determine
 - the initial velocity
 - the maximum height of the ball **T/I A**
- You throw a ball straight up into the air at 18 m/s from a height of 32 m above the ground. **T/I**
 - Calculate the time the ball takes to hit the ground.
 - Calculate the velocity of the ball when it hits the ground.
 - Calculate the maximum height of the ball.
 - Explain why you cannot use half the time for (a) to answer (c).
- A toy rocket starts from rest on the ground and then accelerates at 39.2 m/s^2 [up] for 5.0 s. **T/I**
 - Calculate the velocity of the rocket when the engines stop firing after 5.0 s.
 - Calculate the maximum height of the rocket.
 - How long will it take the rocket to hit the ground from rest at the maximum height and initial rest?
- When a driver is forced to make a panic stop by pressing down on the brake as hard as possible, the car will undergo a large acceleration to stop. **K/U T/I A**
 - Copy **Table 2** into your notebook. Calculate the braking distances.
 - Compare the reaction times, and discuss the implications, to the driver and to speeding, of using cellphones while driving.

Table 2

	Acceleration (m/s^2)	Reaction time (s)	Speed (km/h)	Braking distance (m)
(i)	9.5	0.80	60.0	
(ii)	9.5	0.80	120.0	
(iii)	9.5	2.0	60.0	